MATH 54 - MIDTERM 2 STUDY GUIDE

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Note: 1.3.4 means 'Problem 4 in section 1.3'

IMPORTANT QUESTIONS

Chapter 3: Determinants

- **Calculate the determinant of a matrix, possibly using row-reductions** (3.1.9, 3.1.11, 3.1.13, 3.2.5, 3.2.7, 3.2.11, 3.2.21)
- Calculate volumes using determinants (3.3.21, 3.3.32)

Chapter 4: Vector Spaces

- Determine if a set is a vector space or not (4.1.1, 4.1.3, 4.1.17, 4.2.7, 4.2.9, 4.2.11)
- Given $[\mathbf{x}]_{\mathcal{B}}$, find $\mathbf{x}$, and vice-versa (4.4.1, 4.4.3, 4.4.5, 4.4.7, 4.4.11)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^n$ (4.4.9)
- Determine whether a set is linear independent or dependent (4.3.3, 4.4.27)
- Find a basis and state the dimension of a vector space (4.5.1, 4.5.3, 4.5.5, 4.5.7, 4.5.9, 4.5.11)
- Given a matrix A, find a basis for $Nul(A)$, $Col(A)$, $Row(A)$, and also find $Rank(A)$ (4.2.3, 4.2.5, 4.3.9, 4.3.11, 4.5.13, 4.5.15, 4.5.17, 4.6.1, 4.6.3)
- Use the rank-nullity theorem to find $Rank(A)$ etc. (4.6.7, 4.6.9, 4.6.11, 4.6.13, 4.6.15)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (4.7.7, 4.7.9)
- Use the change-of-coordinates matrix to find $[\mathbf{x}]_{\mathcal{C}}$ $[\mathbf{x}]_{\mathcal{B}}$ (4.7.1, 4.7.3)

Chapter 5: Diagonalization

- **Find a diagonal matrix D and a matrix P such that $A = PDP^{-1}$, or say A is not diagonalizable** (5.2.9, 5.2.11, 5.2.13, 5.3.9, 5.3.11, 5.3.17)
- Show that a given matrix is not diagonalizable (5.4.11)
- **Find the matrix of a linear transformation** (5.4.1, 5.4.3, 5.4.9, 5.4.11, 5.4.17(b))

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Chapter 6: Inner Products and Norms

- Determine if a set is orthogonal, or orthonormal (6.2.3, 6.2.19, 6.2.21)
- Find the orthogonal projection of $\mathbf{x}$ on a subspace W. Use this to write $\mathbf{x}$ as a sum of two orthogonal vectors, and to find the smallest distance between $\mathbf{x}$ and W (6.2.11, 6.3.3, 6.3.5, 6.3.1, 6.3.7, 6.2.15, 6.3.11)
- **Use the Gram-Schmidt process to produce an orthonormal basis of a subspace W spanned by some vectors** (6.4.1, 6.4.3, 6.4.5, 6.4.7, 6.4.9, 6.4.11)
- **Find the least-squares solution (and least-squares error) of an inconsistent system of equations** (6.5.1, 6.5.3, 6.5.7, 6.5.9, 6.5.11)
- Find inner products, lengths, and orthogonal projections of functions f and g using fancier inner products $\langle f, g\rangle$ (6.7.3, 6.7.5, 6.7.7, 6.7.9, 6.7.11, 6.7.22, 6.7.24)
- Show a given formula defines an inner product (6.7.13)
- Use the Gram-Schmidt process to find an orthonormal basis of **functions** (6.7.25, 6.7.26)
- Remember the Cauchy-Schwarz inequality (6.7.19, 6.7.20)

TRUE/FALSE EXTRAVAGANZA

Do the following set of T/F questions: 3.2.27, 4.1.24, 4.2.25, 4.3.21, 4.6.17, 4.7.11, 5.3.21, 6.3.21, 6.5.17 (check out the hints to HW 4,5,6,7,8 for answers)

CONCEPTS

Understand the following concepts:

- Vector space, Subspace (4.1, 4.2)
- Basis, Dimension (4.3, 4.5)
- Coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$ (4.4)
- $Nul(A)$, $Col(A)$, $Row(A)$, Rank (4.6)
- Rank-Nullity Theorem (4.6)
- Change of coordinates matrix (4.7)
- Eigenvalues, Eigenvectors, Characteristic polynomial (5.1 - 5.3)
- A is similar to B (5.2)
- Diagonalizable, Diagonalization Theorem (Theorem 5 in section 5.3)
- Matrix of a Linear transformation (5.4)
- Inner products, Norms, Orthogonal vectors, Orthogonal Matrix (6.1)
- Orthogonal projection (6.2, 6.3)
- Gram-Schmidt process (6.4)
- Least-squares (6.5)
- Inner product space (6.7)
- Cauchy-Schwarz inequality (6.7)

NOT-SO-IMPORTANT QUESTIONS (ONLY DO THEM IF YOU HAVE THE TIME)

- Solve questions using the fact that $det(AB) = det(A)det(B)$ (3.2.31, 3.2.33, 3.2.34, 3.2.35)
- Solve a system using Cramer's rule (3.3.1, 3.3.3, 3.3.5)
- Show the intersection of two subspaces is a subspace (4.1.32)
- Show the nullspace/range of a linear transformation are vector spaces (4.2.30)
- Find an infinite-dimensional vector space (4.5.27)
- Use the decomposition $A = PDP^{-1}$ to find A^k for any k (5.3.1, 5.3.3)
- Find the complex eigenvalues and eigenvectors of a matrix (5.5.1, 5.5.3, 5.5.5)
- Find a matrix C and a matrix P such that $A = PCP^{-1}$ (5.5.13, 5.5.15, 5.5.17)
- Interpret what the matrix C means geometrically (5.5.7, 5.5.9)
- Prove the parallelogram law (6.1.24)
- Given an orthogonal basis $\mathcal{B}$, find $[\mathbf{x}]_{\mathcal{B}}$ (6.2.9)
- Find the QR-decomposition of a matrix A (6.4.13)
- Use the QR-factorization of a matrix A to find a least-squares solution of $A\mathbf{x} = \mathbf{b}$ (6.5.15)
- Use the method of least-squares to fit data points to a line (6.6.1, 6.6.3)
- Same thing as above, but with weighted least squares (6.8.1, 6.8.2)
- Find quadratic/cubic trend functions (6.8.3, 6.8.4)